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Magnetostatic modes in a finite superlattice

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Abstract. We develop a direct method by which we exactly derive a general dispersion relation for magnetostatic modes in an N-cell finite superlattice composed of two different alternating magnetic layers. We also confirm mathematically the consistency condition for the existence of surface modes on a semi-infinite superlattice. We obtain some new analytical conclusions about surface modes.

1. Introduction

During the past decade, the rapid progress in such techniques as molecular beam epitaxy and metal-organic chemical vapour deposition has made possible the extensive investigation of various kinds of multilayers. Recently, magnetic multilayers have attracted significant attention because of their unique magnetic properties. The surface and interfaces in the layered system may profoundly influence the properties of the entire structure. Much of the most interesting physics occurring in the layered system is due to surface or interface effects. Some collective excitations in a layered structure are localized to the interfaces and surfaces, and thus make ideal probes for examining surface and interface conditions. Spin wave excitations in a periodic multilayered system, a superlattice, have been extensively studied for infinite and semi-infinite cases. Camley and co-workers dealt with the magnetic–non-magnetic superlattice in a parallel magnetization geometry where the saturation magnetization lay parallel to the layers (Camley *et al* 1983). The study was extended to the case where the magnetization lies along the normal to the layers (Camley and Cottam 1987). Barnas treated the more general infinite and semi-infinite superlattices, which are composed of N different magnetic materials (Barnas 1988a, b, c). Recently, the calculations done earlier by Camley and Cottam were generalized to the case in which the magnetization is not confined to be either parallel or perpendicular to the layers (Li *et al* 1994). There have also been some theoretical works (Albuquerque *et al* 1991, Barnas 1988a, b, c, Grunberg and Mika 1983, Johnson *et al* 1985) on finite superlattices. The finite superlattices may be more interesting due to the fact that they have more surface effects than semi-infinite structures and because an actual sample made by experiment is a finite-sized structure. What the earlier researchers were most concerned about are the unique surface effects in a finite structure and the approximation of a large finite superlattice by an infinite or a semi-infinite structure (Johnson *et al* 1985). Experimentally, Schuller and Grimsditch (1985) observed the long-wavelength spin-wave collective excitations by using Brillouin light scattering techniques. A complete and detailed review of this field was given by Camley and Stamps (1993), and non-reciprocal surface waves were summarized by Camley (1987).

Infinite superlattices and semi-infinite superlattices can be conveniently treated within a transfer-matrix formalism (Raj and Tilley 1989). By introducing a decay parameter β into the magnetic scalar potential, one can separately treat the bulk and surface modes according to whether β is an imaginary or complex parameter. A real surface mode corresponds to the case where the real part of β is positive; furthermore, β must agree with the consistency requirement as indicated by Camley and Cottam (Camley and Cottam, 1987). In this paper, in order to treat a finite superlattice we develop a direct method by which we can exactly derive the dispersion relation for magnetostatic modes, including both surface and bulk modes, in a finite superlattice consisting of two different materials. Our method is different from the conventional transfer-matrix method because we do not assume a value for the decay parameter β. We deal with each magnetic layer as a magnetic film and then connect the magnetic scalar potential in each layer by employing boundary conditions at all interfaces and surfaces. The key problem here is how to solve the boundary conditions, which seem to be very complex and too numerous. In our method a very simple formalism has been obtained after some algebra. Comparing the spectra for different finite-sized superlattices to the spectra for a corresponding infinite or semi-infinite structure, one can find many unique features of the finite structure and observe the trend of the changes in the spectra for a finite structure as the number of elementary units increases. The comparison is helpful when investigating how many layers are necessary before a superlattice can be adequately modelled by a semi-infinite or infinite structure. We find many remarkable differences in the long-wavelength region of the spectra, even for such a large system as a 50-cell structure.

We also confirm mathematically the consistency condition for surface modes to exist on a semi-infinite superlattice, indicated by Camley and Cottam (1987). Some analytical conclusions about the surface modes have been made for some special magnetization geometries.

In section 2, we discuss the condition for the existence of surface modes on a semi-infinite superlattice and give some analytical results. In section 3, we derive an exact solution for magnetostatic modes in an N-cell finite superlattice formed by alternating two different magnetic layers and give some numerical examples. Finally, some general conclusions are presented in section 4.

2. Condition of surface modes

We consider here the case where the wavelengths of spin waves are so long that the influence of short-range exchange interactions can be neglected but are short enough that

$$(2\pi/\lambda)c >> \omega$$

where λ is the wavelength of the spin wave, ω the frequency and c the speed of light in a vacuum. Under such conditions, the behaviour of the spin waves is governed by the magnetostatic form of the Maxwell equations

$$\nabla \times \boldsymbol{H} = 0 \tag{1}$$

$$\nabla \cdot (\boldsymbol{H} + 4\pi \boldsymbol{M}) = 0 \tag{2}$$

where the field $\boldsymbol{H}$ and the magnetization $\boldsymbol{M}$ can be written as the sum of time-independent and time-dependent components of the form

$$\boldsymbol{H} = \boldsymbol{H}_i + \boldsymbol{h}\mathrm{e}^{-\mathrm{i}\omega t} \tag{3}$$

$$\boldsymbol{M} = \boldsymbol{M}_0 + \boldsymbol{m}\mathrm{e}^{-\mathrm{i}\omega t}. \tag{4}$$

Here h and $b = h + 4\pi m$ also obey the magnetostatic equations. For sufficiently long wavelengths, the dynamic magnetic properties of the system can be described by the constitutive relation

$$m = \chi h \tag{5}$$

where χ is the susceptibility tensor; χ can be determined by solving the Bloch equation of motion

$$\frac{\mathrm{d}M}{\mathrm{d}t} = \gamma M \times H. \tag{6}$$

Here γ is the gyromagnetic ratio.

We take the z axis of a cartesian coordinate system to be along the normal to the surface of the semi-infinite superlattice, which occupies the half-space $z \geqslant 0$. The structure considered consists of two materials labelled by the indices 1 and 2, respectively. The elementary units, with length $L = d_1 + d_2$, are labelled by the integer n and the outermost elementary unit corresponds to $n = 0$. We restrict our discussion to the two cases where the saturation magnetization lies either along the x axis or along the z axis; then, in terms of the magnetic scalar potential Φ defined by $h = -\nabla\Phi$, the magnetostatic equations reduce to

$$\sum_i \mu_{ii} \frac{\partial^2 \Phi}{\partial x_i^2} = 0. \tag{7}$$

Here μ_{ii} are the components of the tensor of magnetic permeability, which is defined by $\mu = I + 4\pi\chi$ and is different for the two magnetization geometries. Following Camley and Cottam (1987), we assume a solution of the form of a plane-wave propagating parallel to the surface:

$$\Phi = \phi(z) \mathrm{e}^{\mathrm{i}(q_x x + q_y y - \omega t)} \tag{8}$$

where

$$\phi(z) = \begin{cases} C\mathrm{e}^{qz} & z \leqslant 0 \\ \mathrm{e}^{-\beta nL}(A_+ \mathrm{e}^{\alpha_1(z-nL)} + A_- \mathrm{e}^{-\alpha_1(z-nL)}) & nL \leqslant z \leqslant nL + d_1 \\ \mathrm{e}^{-\beta nL}(B_+ \mathrm{e}^{\alpha_2(z-nL-d_1)} + B_- \mathrm{e}^{-\alpha_2(z-nL-d_1)}) & nL + d_1 \leqslant z \leqslant (n+1)L \end{cases} \tag{9}$$

with

$$q^2 = q_x^2 + q_y^2$$

and

$$\alpha_{1,2}^2 = (\mu_{xx}^{(1,2)} q_x^2 + \mu_{yy}^{(1,2)} q_y^2)/\mu_{zz}^{(1,2)}. \tag{10}$$

The terms q_x and q_y are the components of the wavevector along the x and y directions, respectively. The coefficients $A_\pm$, $B_\pm$ and C will be determined by applying the boundary conditions. In order to guarantee that the solution we find is a true surface wave, β must satisfy the inequality given by

$$\mathrm{Re}(\beta) > 0. \tag{11}$$

The usual electromagnetic boundary conditions are that the tangential $\boldsymbol{h}$ and normal $\boldsymbol{b}$ components are continuous across all the boundaries. In the geometry considered, the boundary conditions become that Φ and $b_z = -\sum_i \mu_{zi}\partial\Phi/\partial x_i$ are continuous at $z = nL$ and $z = nL + d_1$. The application of the boundary conditions results in

$$\begin{aligned}
&C = A_+ + A_- \\
&qC = \lambda_+^{(1)}A_+ + \lambda_-^{(1)}A_- \\
&\mathrm{e}^{\alpha_1 d_1}A_+ + \mathrm{e}^{-\alpha_1 d_1}A_- = B_+ + B_- \\
&\lambda_+^{(1)}\mathrm{e}^{\alpha_1 d_1}A_+ + \lambda_-^{(1)}\mathrm{e}^{-\alpha_1 d_1}A_- = \lambda_+^{(2)}B_+ + \lambda_-^{(2)}B_- \\
&\mathrm{e}^{-\beta L}(A_+ + A_-) = \mathrm{e}^{\alpha_2 d_2}B_+ + \mathrm{e}^{-\alpha_2 d_2}B_- \\
&\mathrm{e}^{-\beta L}(\lambda_+^{(1)}A_+ + \lambda_-^{(1)}A_-) = \lambda_+^{(2)}\mathrm{e}^{\alpha_2 d_2}B_+ + \lambda_-^{(2)}\mathrm{e}^{-\alpha_2 d_2}B_-
\end{aligned} \tag{12}$$

with

$$\lambda_\pm^{(1,2)} = \pm\alpha_{1,2}\mu_{zz}^{(1,2)} + \mathrm{i}(\mu_{zx}^{(1,2)}q_x + \mu_{zy}^{(1,2)}q_y).$$

Eliminating B_+, B_- and C from equation (12), one can obtain a set of three linear homogeneous equations in two unknowns, A_+ and A_-, as follows:

$$(\lambda_+^{(1)} - \lambda_+^{(2)})(1 - \mathrm{e}^{\beta L + \alpha_1 d_1 - \alpha_2 d_2})A_+ + (\lambda_-^{(1)} - \lambda_+^{(2)})(1 - \mathrm{e}^{\beta L - \alpha_1 d_1 - \alpha_2 d_2})A_- = 0 \tag{13a}$$

$$(\lambda_+^{(1)} - \lambda_-^{(2)})(1 - \mathrm{e}^{\beta L + \alpha_1 d_1 + \alpha_2 d_2})A_+ + (\lambda_-^{(1)} - \lambda_-^{(2)})(1 - \mathrm{e}^{\beta L - \alpha_1 d_1 + \alpha_2 d_2})A_- = 0 \tag{13b}$$

$$(\lambda_+^{(1)} - q)A_+ + (\lambda_-^{(1)} - q)A_- = 0 \tag{13c}$$

with β a parameter. A system of linear homogeneous equations has a non-trivial solution only if the rank of the coefficient matrix is smaller than the number of unknowns. In the case under consideration, the number of unknowns is two, and so for a non-trivial solution the rank of the coefficient matrix must be one, which implies the determinants of the three 2×2 submatrixes in the coefficient matrix all vanish. For a magnetic–non-magnetic semi-infinite superlattice, this condition becomes

$$\begin{aligned}
(\lambda_+^{(1)} - q)(\lambda_-^{(1)} + q)&(1 - \mathrm{e}^{\beta L + \alpha_1 d_1 - q d_2})(1 - \mathrm{e}^{\beta L - \alpha_1 d_1 + q d_2}) \\
&- (\lambda_+^{(1)} + q)(\lambda_-^{(1)} - q)(1 - \mathrm{e}^{\beta L - \alpha_1 d_1 - q d_2})(1 - \mathrm{e}^{\beta L + \alpha_1 d_1 + q d_2}) = 0
\end{aligned} \tag{14a}$$

$$(\lambda_+^{(1)} + q)(\lambda_-^{(1)} - q)(1 - \mathrm{e}^{\beta L + \alpha_1 d_1 + q d_2}) - (\lambda_-^{(1)} + q)(\lambda_+^{(1)} - q)(1 - \mathrm{e}^{\beta L - \alpha_1 d_1 + q d_2}) = 0 \tag{14b}$$

$$(\lambda_+^{(1)} - q)(\lambda_-^{(1)} - q)\sinh(\alpha_1 d_1) = 0. \tag{14c}$$

Here β should satisfy all three equations at the same time. According to Camley and Stamps (1993), equation (14c) has three possible cases: $\lambda_+^{(1)} - q = 0$, $\lambda_-^{(1)} - q = 0$ and $\alpha_1 d_1 = \mathrm{i}m\pi$, where $m = 0, \pm 1, \pm 2, \cdots$. Combining (14a) and (14c), we have

$$\begin{aligned}
&\lambda_+^{(1)} - q = 0 \qquad && \beta L = \pm(\alpha_1 d_1 + q d_2) \\
&\lambda_-^{(1)} - q = 0 \qquad && \beta L = \pm(\alpha_1 d_1 - q d_2) \\
&\alpha_1 d_1 = \mathrm{i}m\pi \qquad && \beta L = \pm q d_2 + \mathrm{i}(2m+1)\pi.
\end{aligned}$$

Similarly, combining (14b) and (14c), we obtain

$$\lambda_+^{(1)} - q = 0 \qquad \beta L = -(\alpha_1 d_1 + q d_2)$$
$$\lambda_-^{(1)} - q = 0 \qquad \beta L = \alpha_1 d_1 - q d_2$$
$$\alpha_1 d_1 = \mathrm{i} m\pi \qquad \beta L = -q d_2 + \mathrm{i}(2m+1)\pi.$$

Obviously, the three consistency solutions for β, which simultaneously satisfy (14a)–(14c), are

$$\beta L = -(\alpha_1 d_1 + q d_2)$$
$$\beta L = \alpha_1 d_1 - q d_2$$
$$\beta L = -q d_2 + \mathrm{i}(2m+1)\pi.$$

After ignoring a solution that does not represent a true surface mode due to $\mathrm{Re}(\beta) < 0$, we only have the solution $\beta L = \alpha_1 d_1 - q d_2$ with $\lambda_-^{(1)} - q = 0$. When α_1 is real and $\alpha_1 d_1 - q d_2 > 0$, this solution represents the type of surface modes which are composed of surface waves in each magnetic layer. Obviously, the other type of surface modes, which consists of bulk waves in each magnetic film, cannot exist on a magnetic–non-magnetic semi-infinite superlattice due to the consistency requirement. Mathematically we confirm the conclusion made by Camley and Cottam (1987) and explain the differences between earlier researchers in conclusions about surface modes (Camley *et al* 1983, Camley and Cottam 1987, Shen and Li 1992, Camley and Stamps 1993).

We now discuss the possibility of surface waves existing on a semi-infinite superlattice composed of two arbitrary materials. We consider three special cases: (i) $\boldsymbol{H}_0$ parallel to the x direction and $q_x = 0$; (ii) $\boldsymbol{H}_0$ parallel to the x direction and $q_y = 0$; (iii) $\boldsymbol{H}_0$ parallel to the z direction. According to Villeret *et al* (1989), in cases (i) and (ii) $\mu_{xx}^{(1,2)} = 1$, $\mu_{yy}^{(1,2)} = \mu_{zz}^{(1,2)} = \mu_{1,2}$, $\chi_{zx}^{(1,2)} = 0$ and $\chi_{zy}^{(1,2)} = \chi_{1,2}$, while in case (iii) $\mu_{zz}^{(1,2)} = 1$, $\mu_{xx}^{(1,2)} = \mu_{yy}^{(1,2)} = \mu_{1,2}$, and $\chi_{zx}^{(1,2)} = \chi_{zy}^{(1,2)} = 0$. Using these relations and combining (13b) and (13c), we have

$$\mathrm{e}^{-\beta L} = \mathrm{e}^{\alpha_2 d_2}[f \sinh(\alpha_1 d_1) + \cosh(\alpha_1 d_1)]$$

with

$$f = \begin{cases} [4\pi\mathrm{i}(\chi_1 - \chi_2) + \mu_1]/\mu_1 & \text{for case (i)} \\ (\mu_1 + \sqrt{\mu_2})/[\sqrt{\mu_1}(1 + \sqrt{\mu_2})] & \text{for cases (ii) and (iii)} \end{cases}$$

and

$$\alpha_{1,2} = \begin{cases} q & \text{for case (i)} \\ q/\sqrt{\mu_{1,2}} & \text{for case (ii)} \\ q\sqrt{\mu_{1,2}} & \text{for case (iii).} \end{cases}$$

Obviously, the decay condition, $\mathrm{Re}(\beta) > 0$, for a surface mode to exist on a semi-infinite superlattice becomes $f < 0$ when $\mu_{1,2} > 0$. One can easily find that the condition cannot hold for cases (ii) and (iii) due to $f > 0$. Therefore the surface modes cannot exist in the two cases. This conclusion is different from earlier numerical results (Shen and Li 1992).

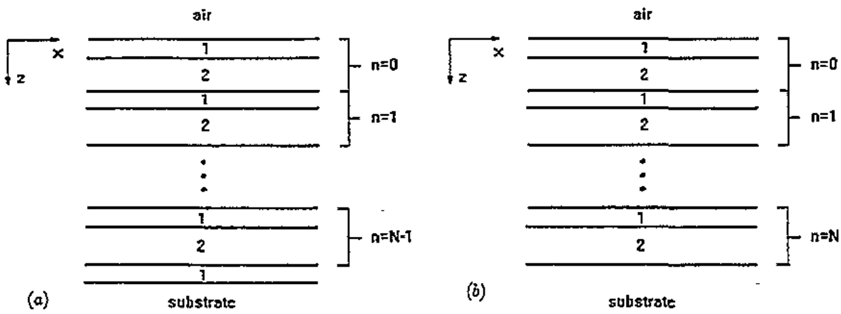


Figure 1. The two different geometries of a finite superlattice composed of alternating layers of materials labelled by 1 and 2, respectively. The z axis is taken to be along the normal to the surface. The elementary units are indexed by n, as shown. In (a) the bottom layer is composed of material 1; in (b) the bottom layer is composed of material 2.

3. Finite superlattices

We now deal with the problem of magnetostatic modes in the finite superlattices formed by different alternating magnetic layers. In the case considered, there are two possible structures in which the outermost layers are either composed of the same material or composed of different materials, as shown in figures 1(a) and (b), respectively. The finite superlattice, assumed to have a non-magnetic substrate, is confined in the region $0 \leqslant z \leqslant NL + d_1$ for the first case and in the region $0 \leqslant z \leqslant (N+1)L$ for the second case, respectively. The first structure may be more interesting physically, because it has high symmetry. Instead of introducing the decay parameter β, we let $\phi(z)$ in (8) be of the form

$$\phi(z) = \begin{cases} C\mathrm{e}^{qz} & z \leqslant 0 \\ A_n^{(+)}\mathrm{e}^{\alpha_1(z-nL)} + A_n^{(-)}\mathrm{e}^{-\alpha_1(z-nL)} & nL \leqslant z \leqslant nL + d_1 \\ B_n^{(+)}\mathrm{e}^{\alpha_2(z-nL-d_1)} + B_n^{(-)}\mathrm{e}^{-\alpha_2(z-nL-d_1)} & nL + d_1 \leqslant z \leqslant (n+1)L \\ D\mathrm{e}^{-q(z-NL-d_1)} & z \geqslant NL + d_1 \end{cases} \tag{15}$$

where $n = 0, 1, \cdots, N-1$. For the first structure, applying the boundary conditions at interfaces $z = nL + d_1$ and $z = (n+1)L$, we obtain

$$\begin{aligned} &\mathrm{e}^{\alpha_1 d_1} A_n^{(+)} + \mathrm{e}^{-\alpha_1 d_1} A_n^{(-)} = B_n^{(+)} + B_n^{(-)} \\ &\lambda_+^{(1)}\mathrm{e}^{\alpha_1 d_1} A_n^{(+)} + \lambda_-^{(1)}\mathrm{e}^{-\alpha_1 d_1} A_n^{(-)} = \lambda_+^{(2)} B_n^{(+)} + \lambda_-^{(2)} B_n^{(-)} \\ &A_{n+1}^{(+)} + A_{n+1}^{(-)} = \mathrm{e}^{\alpha_2 d_2} B_n^{(+)} + \mathrm{e}^{-\alpha_2 d_2} B_n^{(-)} \\ &\lambda_+^{(1)} A_{n+1}^{(+)} + \lambda_-^{(1)} A_{n+1}^{(-)} = \lambda_+^{(2)}\mathrm{e}^{\alpha_2 d_2} B_n^{(+)} + \lambda_-^{(2)}\mathrm{e}^{-\alpha_2 d_2} B_n^{(-)}. \end{aligned} \tag{16}$$

The application of the boundary conditions at the top and bottom surfaces results in

$$\begin{aligned} &C = A_0^{(+)} + A_0^{(-)} \\ &qC = \lambda_+^{(1)} A_0^{(+)} + \lambda_-^{(1)} A_0^{(-)} \\ &D = \mathrm{e}^{\alpha_1 d_1} A_N^{(+)} + \mathrm{e}^{-\alpha_1 d_1} A_N^{(-)} \\ &-qD = \lambda_+^{(1)}\mathrm{e}^{\alpha_1 d_1} A_N^{(+)} + \lambda_-^{(1)}\mathrm{e}^{-\alpha_1 d_1} A_N^{(-)}. \end{aligned} \tag{17}$$

After eliminating $B_n^{(+)}$ and $B_n^{(-)}$ from (16) and eliminating C and D from (17), we have

$$A_N^{(-)}/A_N^{(+)} = -(\lambda_+^{(1)} + q)e^{2\alpha_1 d_1}/(\lambda_-^{(1)} + q) \tag{18}$$

$$A_0^{(-)}/A_0^{(+)} = -(\lambda_+^{(1)} - q)/(\lambda_-^{(1)} - q) \tag{19}$$

$$A_{n+1}^{(-)}/A_{n+1}^{(+)} = [Q'(A_n^{(-)}/A_n^{(+)}) + R']/[Q + R(A_n^{(-)}/A_n^{(+)})] \tag{20}$$

where

$$Q = \left[(\lambda_+^{(1)} - \lambda_+^{(2)})(\lambda_-^{(1)} - \lambda_-^{(2)})e^{-\alpha_2 d_2} - (\lambda_+^{(1)} - \lambda_-^{(2)})(\lambda_-^{(1)} - \lambda_+^{(2)})e^{\alpha_2 d_2}\right]e^{\alpha_1 d_1}$$

$$R = (\lambda_-^{(1)} - \lambda_+^{(2)})(\lambda_-^{(1)} - \lambda_-^{(2)})(e^{-\alpha_2 d_2} - e^{\alpha_2 d_2})e^{-\alpha_1 d_1}$$

$$Q' = \left[(\lambda_+^{(1)} - \lambda_+^{(2)})(\lambda_-^{(1)} - \lambda_-^{(2)})e^{\alpha_2 d_2} - (\lambda_+^{(1)} - \lambda_-^{(2)})(\lambda_-^{(1)} - \lambda_+^{(2)})e^{-\alpha_2 d_2}\right]e^{-\alpha_1 d_1}$$

$$R' = (\lambda_+^{(1)} - \lambda_+^{(2)})(\lambda_+^{(1)} - \lambda_-^{(2)})(e^{\alpha_2 d_2} - e^{-\alpha_2 d_2})e^{\alpha_1 d_1}.$$

By combining (18)–(20) and defining a function as follows

$$F(x) = \frac{Q' + R'/x}{Q + Rx}$$

we have

$$-(\lambda_+^{(1)} + q)e^{2\alpha_1 d_1}/(\lambda_-^{(1)} + q) = \underbrace{t \cdot F(t) \cdot F[t \cdot F(t)] \cdots}_{N \text{ factors}} \tag{21}$$

where

$$t = \frac{R'(\lambda_-^{(1)} - q) - Q'(\lambda_+^{(1)} - q)}{Q(\lambda_-^{(1)} - q) - R(\lambda_+^{(1)} - q)}.$$

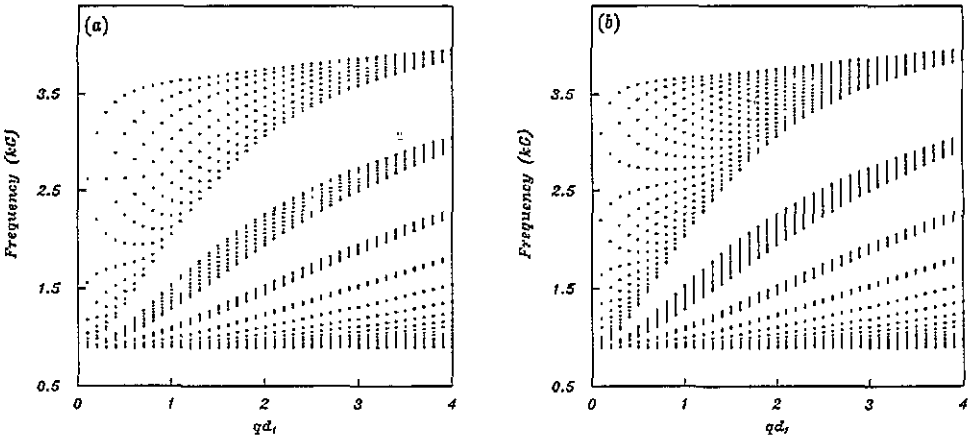
Equation (21) is the implicit dispersion relation for the magnetostatic modes in a finite superlattice, the solutions of which give the dispersion relation for bulk and surface modes at the same time. Taking the Fe superlattice as an example, we try to investigate quantitatively the difference between a finite and an infinite superlattice in their spectra of spin-wave excitations. For simplicity, we consider a perpendicular magnetization geometry. In this situation, the non-vanishing components of the tensor of magnetic permeability are

$$\mu_{xx}^{(1)} = \mu_{yy}^{(1)} = 1 + (4\pi H_i M_0)\left[H_i^2 - (\omega/\gamma)^2\right]^{-1}$$

$$\mu_{xy}^{(1)} = -\mu_{yx}^{(1)} = 4\pi i M_0(\omega/\gamma)\left[H_i^2 - (\omega/\gamma)^2\right]^{-1}$$

$$\mu_{zz}^{(1)} = \mu_{xx}^{(2)} = \mu_{yy}^{(2)} = \mu_{zz}^{(2)} = 1$$

where $H_i = H_0 - 4\pi M_0$. The parameters appropriate for Fe are $M_0 = 1.68$ KG, $H_0 = 22$ KG, assuming $d_1/d_2 = 0.5$. Figures 2(*a*)–(*c*) present the dispersion curves for three different finite-sized superlattices, with $N = 10$, 20, 50, respectively. Here, for simplicity, we only take 40 discrete values of qd_1 in numerical calculations, though they should be continuous. Using earlier researchers' results (Camley and Cottam 1987), we add the boundaries of the bulk bands for a corresponding infinite structure with the same



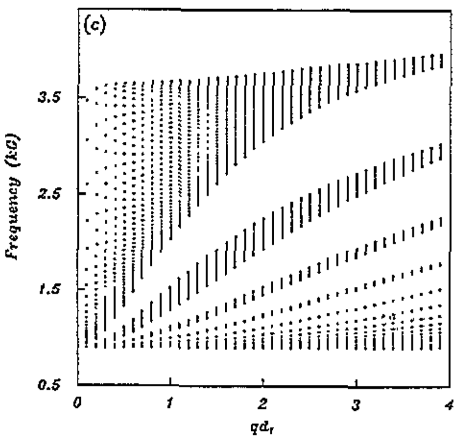


Figure 2. Frequency against dimensionless wavevector qd_1 (d_1 held constant) for different finite-sized Fe superlattices. The parameters used are $M_0 = 1.68$ KG, $H_0 = 22$ KG and $d_2/d_1 = 0.5$. In (*a*) $N = 10$; in (*b*) $N = 20$; in (*c*) $N = 50$.

parameters as these figures, so as to compare the finite superlattice with the infinite or semi-infinite structure. Unlike the infinite structure, the spectra for finite structures are characterized by discrete modes. The reason for this, of course, is that the periodicity is broken and Bloch's theorem does not hold in the finite structure. As pointed out by earlier researchers (Camley and Stamps 1993), there are as many modes as magnetic layers. The structures in the spectra are obviously different for different finite-sized superlattices, particularly in the long-wavelength region. However, one finds the outlines of the spectra are very close to the boundaries of the bulk bands when the number of elementary units, N, is larger than 20. This supports a conclusion made for other systems (Johnson *et al* 1985).

There has been a discussion about how many layers are necessary before a superlattice can be adequately modelled by a semi-infinite or infinite structure. If the 50-cell superlattice can be considered a large enough system, one can avoid a detailed consideration of the surfaces by imposing periodic boundary conditions on the finite structure and subsequently pass to the limit in which the periodic volume becomes infinite (Callaway 1991). In this way, the dispersion relation is given by (Camley and Cottam 1987)

$$\cos(QL) = \cosh(\alpha_1 d_1)\cosh(qd_2) + \tfrac{1}{2}(\alpha_1/q + q/\alpha_1)\sinh(\alpha_1 d_1)\sinh(qd_2)$$

where $QL = 2\pi m/N$, with $N = 50$ and $m = 0, 1, \cdots, 49$. Figure 3 presents the dispersion curves for the infinite structure. A comparison of figure 2(*c*) with figure 3 shows some remarkable differences.

(i) There are 25 modes for the infinite structure formed by imposing periodic boundary conditions on the 50-cell superlattice because of the degeneracy. But the real 50-cell structure has 50 modes. This implies that surface effects make the degeneracy disappear.

(ii) The differences become small when $qd_1 > 1$. This means that, for the size considered, Bloch's theorem approximately holds in the short-wavelength region despite the non-existence of periodicity. However, it is obvious that the structures of the two spectra are very different from each other in the long-wavelength region. Therefore, N must be much larger than 50 so as to obtain a good approximation for these long-wavelength modes in the case considered.

The spectrum of the finite superlattice will be changed if the outermost units are different from those inside the structure. When the top and bottom layers are symmetrically added, with the same additional thickness δ (here $\delta/d_1 = 0.25$), surface modes will appear as shown in figure 4. The dispersion curves for the finite superlattice are very similar to those for a semi-infinite superlattice that has an additional thickness on the top layer (Camley and Cottam 1987), except that the spectrum for the finite structure is discrete. This reflects some similarities between the two structures in the aspects of symmetry and surface properties. Therefore changing the outermost units can be used to control the surface modes.

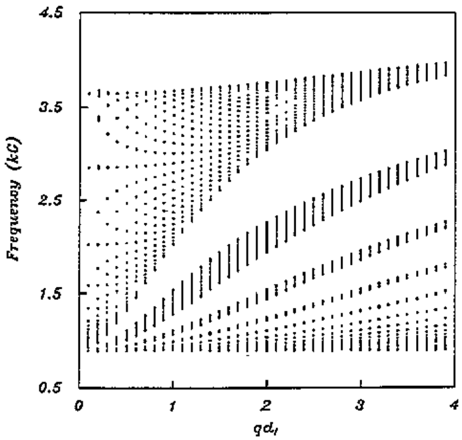


Figure 3. Frequency against dimensionless wavevector qd_1 (d_1 held constant) for the corresponding infinite Fe superlattice formed by imposing periodic boundary conditions on a 50-cell Fe superlattice. Numerical parameters as in figure 2.

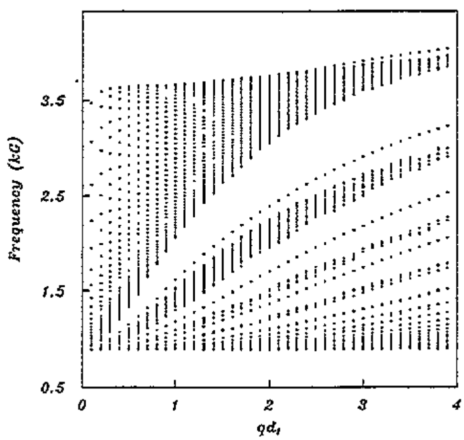


Figure 4. Frequency against dimensionless wavevector qd_1 (d_1 held constant) for a 50-cell Fe superlattice with an additional thickness δ ($\delta/d_1 = 0.25$) on the top and bottom layers of the structure described in figure 1(a). Numerical parameters as in figure 2.

In a more realistic case, the top and bottom layers are composed of different materials as described in figure 1(b). One can easily derive that the changes in boundary conditions require the right-hand side of (18) and the left-hand side of (21) are replaced by

$$-\frac{(\lambda_-^{(2)}+q)(\lambda_+^{(2)}-\lambda_+^{(1)})e^{-\alpha_2 d_2}-(\lambda_+^{(2)}+q)(\lambda_-^{(2)}-\lambda_+^{(1)})e^{\alpha_2 d_2}}{(\lambda_-^{(2)}+q)(\lambda_+^{(2)}-\lambda_-^{(1)})e^{-\alpha_2 d_2}-(\lambda_+^{(2)}+q)(\lambda_-^{(2)}-\lambda_-^{(1)})e^{\alpha_2 d_2}}e^{2\alpha_1 d_1}$$

while the other equations remain unchanged. It has been shown that this change in the bottom unit only causes small differences in the spectra when $N > 10$. Figure 5 displays a numerical example of this structure. The system considered is

$Cd_{0.30}Mn_{0.70}Te$–$Cd_{0.89}Mn_{0.11}Te$, an antiferromagnetic–paramagnetic superlattice at liquid-helium temperatures (Villeret *et al* 1989). The parameters used here are: (i) the sublattice saturation magnetization $M_0^{(1)} = 0.2$ KG; (ii) for the paramagnetic layers $M_0^{(2)} = 0.07$ KG; (iii) the exchange field $H_E = 200$ KG; (iv) the anisotropy field $H_A = 30$ KG; (v) the applied magnetic field $H_0 = 60$ KG, again being perpendicular to the layers; (vi) the number of the elementary units $N = 50$; and (vii) the ratio of the thickness $d_1/d_2 = 2$. From figure 5 one can see that there are three frequency regions in which bulk bands exist. This agrees with the earlier work on a corresponding infinite structure (Villeret *et al* 1989). The fact that no surface modes appear in the numerical results could be a demonstration of the conclusion, made in section 2, that no surface modes can exist in the case where magnetization is perpendicular to the layers.

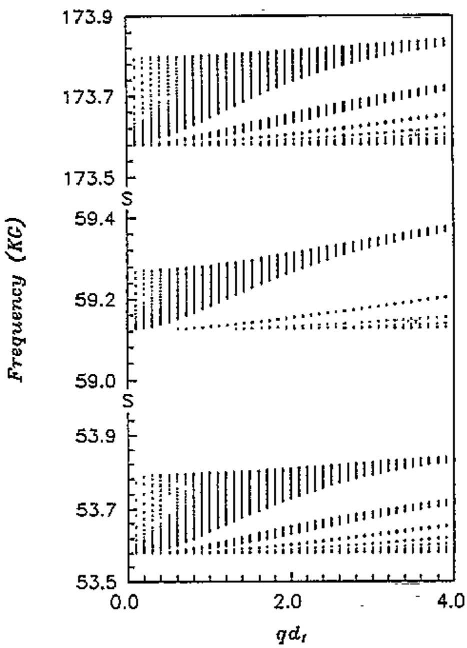


Figure 5. Frequency against dimensionless wavevector qd_1(d_1 held constant) for a 50-cell CdMnTe superlattice. Parameters used are $H_0 = 60$ KG, $H_E = 200$ KG, $H_A = 30$ KG, $M_0^{(1)} = 0.2$ KG, $M_0^{(2)} = 0.07$ KG and $d_2/d_1 = 0.5$.

4. Summary

We have discussed the consistency condition and the decay condition for surface modes to exist on a semi-infinite superlattice. Through an analytical discussion, for perpendicular and parallel magnetization geometries, we arrived at the following analytical conclusions.

(i) On a semi-infinite magnetic–non-magnetic superlattice, those surface modes that are composed of bulk waves from each magnetic film cannot exist because of restrictions from the consistency condition.

(ii) On a semi-infinite superlattice consisting of two arbitrary materials, those surface modes that are composed of surface waves on each magnetic layer cannot exist in the cases with H_0 parallel to the z axis, and H_0 parallel to the x axis with $q_y = 0$.

Without assuming a decay parameter β, we developed a direct method by which one can exactly derive the dispersion relation for magnetostatic modes in an arbitrary N-cell superlattice composed of two materials. The numerical examples of these dispersion relations for different finite-sized superlattices show very interesting discrete structures in the small qd_1 region. When $N > 20$ the outlines of the spectra are very close to the boundary of the bulk bands for a corresponding infinite structure. These facts are consistent with earlier results about plasmons in finite superlattices (Johnson *et al* 1985). We also discussed the approximation of a large finite-size superlattice by an infinite structure through imposing periodic boundary conditions on a 50-cell structure. Comparing the two spectra one can clearly see where Bloch's theorem breaks down as an infinite superlattice changes into a finite superlattice. We find that the surface effects are still significant for the long-wavelength modes even in a 50-cell superlattice. This fact, that the deviation from periodicity has a stronger influence on long-wavelength modes than on short-wavelength modes, seems to be consistent with what one expects for the size effect.

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